Sesión Especial 11

Estructuras geométricas aplicadas a mecánica clásica, teoría de control e ingeniería. / Geometric structures applied to classical mechanics, control theory and applications. Actividad de la Red de Geometría, Mecánica y Control.

Organizadores

- María Barbero Liñán (Universidad Politécnica de Madrid-ICMAT)
- Leonardo Colombo (Instituto de Ciencias Matemáticas (CSIC-UAM-UC3M-UCM))

Descripción

En 1972 David Mayne y Roger Brockett organizaron una conferencia en el Imperial College of London para explicar a jóvenes investigadores las herramientas e ideas de geometría diferencial que serían de gran aplicabilidad en un sinfín de problemas ingenieriles. Esta iniciativa impulsó la investigación en control geométrico y su carácter interdisciplinar. El uso de herramientas típicas de geometría diferencial, topología y teoría de álgebra de Lie ha ayudado a profundizar en el conocimiento de teoría de control geométrico y mecánica geométrica, tanto en el mundo continuo como en el discreto, tanto en mecánica clásica como en cuántica.

La sesión ha reunido expertos nacionales e internacionales en control geométrico y mecánica geométrica que cubren aspectos teóricos y prácticos con el objetivo de identificar nuevas líneas de investigación futura entre matemáticos, físicos e ingenieros y promover el diálogo entre ellos.

Programa

LUNES, 4 de febrero (mañana)

11:30 – 12:00	Manuel de León (ICMAT(CSIC-UAM-UC3M-UCM), Spain) *Contact hamiltonian systems.*
12:00 – 12:30	Cédric M. Campos (Universidad Yachay Tech, Ecuador) *Towards a canonical Hamiltonian formalism for high-order Classical Field Theory.*
12:30 – 13:00	Silvia Vilariño (Centro Universitario de la Defensa, Zaragoza, Spain) *New results on Lie systems.*
13:00 – 13:30	Luis García Naranjo (Universidad Nacional Autónoma de México) *Generalisation of Chaplygin's reducing multiplier theorem and applications.*

Lunes, 4 de febrero (tarde)

17:00 – 17:30	Ignacio Romero (Instituto IMDEA Materiales-Universidad Politécnica de Madrid, Spain) *Thermodynamically consistent integrators for nonlinear solid and fluid mechanics.*
17:30 – 18:00	David Martín de Diego (ICMAT(CSIC-UAM-UC3M-UCM), Spain) *Geometric integration of forced Lagrangian dynamics preserving geometry.*
18:00 – 18:30	Fernando Jiménez (University of Oxford, UK) *A Fractional Variational Approach for Modelling Dissipative Mechanical Systems: Continuous and Discrete Settings.*
18:30 – 19:00	Héctor García de Marina (University of Southern Denmark) *Geometric and numerical methods for formation control of multi-agent control systems.*

Martes, 5 de febrero (mañana)

11:30 – 12:00	François Gay Balmaz (CNRS-École Normale Supérieure de Paris, France) *Dirac structures in nonequilibrium thermodynamics.*
12:00 – 12:30	Marta Zoppello (Universitá degli studi di Padova, Italy) *Microswimmer robots and their control.*
12:30 – 13:00	Olivier Cots (Institut de Recherche en Informatique de Toulouse, France) *Time minimal saturation of a pair of spins and application in Magnetic Resonance Imaging.*
13:00 – 13:30	Mario Sigalotti (Inria, Center de Recherche Paris, France) *Lyapunov exponents of switched systems with dwell time: a geometric approach.*

Contact hamiltonian systems

Manuel de León

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Abstract. In this talk, we will present some recent results on the properties of contact hamiltonian systems. The dynamics of these systems is very different from the one of symplectic systems, and allows us to study dissipative systems and thermodynamics in a very natural geometric framework.

Towards a canonical Hamiltonian formalism for high-order Classical Field Theory

Cédric M. Campos

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Abstract. The Lagrangian and Hamiltonian visions of Classical Field Theories are well understood, although some geometric constructions like Tulczyjew's Triple are recent. However, a long lasting problem where only partial results have been obtained is the proper definition of a non-ambiguous Hamiltonian formalism for systems of order higher than 2. We will give geometric indications on how to solve this problem.

New results on Lie systems

Silvia Vilariño

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Abstract. A Lie system is a non-autonomous system of first-order differential equations describing the integral curves of a non-autonomous vector field taking values in a finite-dimensional Lie algebra of vector fields, a so-called Vessiot–Guldberg Lie algebra. This work analyses Lie systems admitting a Vessiot–Guldberg Lie algebra of Hamiltonian vector fields relative to a several geometric structures. We presents the recents results about this topic.

Generalisation of Chaplygin's reducing multiplier theorem and applications

Luis García Naranjo

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Abstract. One of the most famous theorems in nonholonomic mechanics is Chaplygin's reducing multiplier theorem that gives sufficient conditions under which the equations of motion of a nonholonomic system may be transformed into Hamiltonian form by a time reparametrization. In this talk I will present a generalisation of the conditions given by Chaplygin and I will give applications to the study of multi-dimensional nonholonomic systems.

Thermodynamically consistent integrators for nonlinear solid and fluid mechanics

Ignacio Romero

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Abstract. Energy-Entropy-Momentum (EMM) integration algorithms are a class of second order, implicit methods that preserve, exactly, the laws of thermodynamics when applied to mechanical systems, as well as certain of the symmetries of their equations.
Built on the GENERIC description of the evolution equations, these methods were originally developed for smooth ordinary differential equations. In recent years, however, they have been extended and now can encompass very general problems in solid and fluid mechanics, including finite strain thermoelasticity, solidification, and compressible fluid flow.
In this talk we will describe the ideas behind the formulation of EEM integrators, and their application to selected problems. In addition to verifying their structure preservation ability, we shall demonstrate their remarkable robustness and stability for short and long time simulations.

Joint work with D. Portillo, Univesidad Politécnica de Madrid, Spain.

Geometric integration of forced Lagrangian dynamics preserving geometry.

David Martín de Diego

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Abstract. In this talk we will discuss geometric integration of forced Lagrangian systems. Some of these systems have additional geometric features and we will study if it is possible to preserve some of these properties using geometric integrators like variational integrators or discrete gradient methods. We will pay special attention to forced Euler-Poincaré and Lie-Poisson systems, and the cases of interest are, for instance, when the coadjoint orbits remain invariant, but on them the energy is decreasing along orbits (double bracket dissipation) and, also the case of metriplectic systems where the energy is preserved and the entropy is increasing along the motion.

A Fractional Variational Approach for Modelling Dissipative Mechanical Systems: Continuous and Discrete Settings

Fernando Jiménez

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Abstract. Employing a phase space which includes the (Riemann-Liouville) fractional derivative of curves evolving on real space, we develop a restricted variational principle for Lagrangian systems yielding the so-called restricted fractional Euler-Lagrange equations (both in the continuous and discrete settings), which, as we show, are invariant under linear change of variables. This principle relies on a particular restriction upon the admissible variation of the curves. In the case of the half-derivative and mechanical Lagrangians, i.e. kinetic minus potential energy, the restricted fractional Euler-Lagrange equations model a dissipative system in both directions of time, summing up to a set of equations that is invariant under time reversal. Finally, we show that the discrete equations are a meaningful discretization of the continuous ones.

Geometric and numerical methods for formation control of multi-agent control systems

Héctor García de Marina

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Abstract. Despite the ubiquity of robots, lack of systematic approaches for swarms of robots prevents their full use in real-world environments. Geometric control and in particular the geometric structures of systems evolving on Lie groups, provide systematic methodologies to attack some of the challenges in this problem. During this talk, we will discuss two tools from techniques of geometric mechanics applied to control theory that help the formation control of multi-agent systems. First, an exciting family of geometric integrators, called variational integrators, facilitates the computational requirements for the simulation of massive multi-agent systems. Second, by exploiting the role of left-invariant vector fields in a nonlinear observability problem we can understand from a geometric coordinate free framework the relative localization in multi-agents systems.

Dirac structures in nonequilibrium thermodynamics

François Gay Balmaz

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Abstract. Dirac structures are geometric objects that generalize both Poisson structures and presymplectic structures on manifolds. They naturally appear in the formulation of constrained mechanical systems. In this talk, we show that the evolution equations for nonequilibrium thermodynamics admit an intrinsic formulation in terms of Dirac structures, both on the Lagrangian and the Hamiltonian settings. In absence of irreversible processes these Dirac structures reduce to canonical Dirac structures associated to canonical symplectic forms on phase spaces. This geometric formulation of nonequilibrium thermodynamic thus consistently extends the geometric formulation of mechanics, to which it reduces in absence of irreversible processes. The Dirac structures are associated to a variational formulation of nonequilibrium thermodynamics that we developed earlier and are induced from a nonlinear nonholonomic constraint given by the expression of the entropy production of the system. Several examples will be presented.

Microswimmer robots and their control.

Marta Zoppello

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Abstract. In recent years it has been shown that low Reynolds number swimming can be considered as a control problem which is linear in the control, and without drift. We focus on the system of the N-link swimmer, a generalization of the classical Purcell swimmer [5], that was introduced in [1]. One of the main difficulties in exploiting Control Theory in order to solve effectively motion planning or control problems is the complexity of the hydrodynamic forces exerted by the fluid on the swimmer as a reaction to its shape changes. Whereas the N-link swimmer model introduced is simplified and explicit since Resistive Force Theory is used to couple the fluid and the swimmer. After proving that the swimmer is controllable in the whole plane when it is composed by more than 3 links, we address the minimum time problem to reach a target position. Moreover adding elasticity and magnetism we present a model for a magnetically driven slender micro-swimmer, mimicking a sperm cell (see [4]) . The micro-swimmer can be described by a driftless affine control system where the control is an external magnetic field. Moreover we discuss through numerical simulations how to realize different kind of paths.

Referencias

[1] F. Alouges, A. DeSimone, L. Giraldi, and M. Zoppello. Self-propulsion of slender microswimmers by curvature control: N-link swimmers. *International Journal of Non-Linear Mechanics*, 56, 132–141, (2013).

[2] L. Giraldi, P. Martinon, M. Zoppello. Controllability and Optimal Strokes for N-link Microswimmer. Preprint in Hal (hal-00798363), CDC 2013.

[3] L. Giraldi, P. Martinon, M. Zoppello. Optimal Design of the Three-link Purcell Swimmer. *Phys. Rev. E* 91, 023012 (2015).

[4] F. Alouges, A. DeSimone, L. Giraldi, and M. Zoppello. Can magnetic multilayers propel micro-swimmers mimicking sperm cells? *Soft Robotics*, 2(3):117–128, (2015).

[5] E. M. Purcell: Life at low Reynolds number. *American Journal of Physics*, 45 (1977), 3–11.

Time minimal saturation of a pair of spins and application in Magnetic Resonance Imaging

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Abstract. We discuss the time minimal saturation of a pair of spins of the same species but with inhomogeneities on the applied RF-magnetic field, in relation with the contrast problem in Magnetic Resonance Imaging. We present a complete theoretical analysis based on geometric control to classify the optimal syntheses in the single spin case. For a pair of spins, the analysis is more intricate and the use of numerical methods becomes necessary. The Bocop software (based on direct approach) is used to determine local minimizers for physical test cases and Linear Matrix Inequalities approach is applied to estimate the global optimal value and validate the previous computations. This is complemented by numerical computations combining shooting and continuation methods implemented in the HamPath software to analyze the structure of the time minimal solution with respect to the set of parameters of the species. Besides, symbolic computations techniques are used to handle the singularity analysis and pave the way to a better understanding of the complex optimal structures present in the time minimal saturation of a pair of spins.

Lyapunov exponents of switched systems with dwell time: a geometric approach

Mario Sigalotti

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Abstract. In this talk we show how geometric constructions can be used to investigate the stability of switched systems. The switched paradigm is used in automatic control theory to model dynamical systems with time-depending parameters whose variation can be abrupt. An important goal is to characterize the uniform stability of such systems uniformly with respect to the class of switching signals. Using an extension of the notion of control sets to the case of dwell-time switching signals and exploiting some new results on the induced projective action of the orbit of left-invariant systems on the general linear group, we prove that the Lyapunov exponent can be characterized using only trajectories with periodic angular component.
This extends previous works by F. Colonius and W. Kliemann, both by allowing positive dwell-time and by do not requiring any Lie bracket generating assumption.

Joint work with Francesco Boarotto.
